# 524. The Vapour Pressures of Concentrated Aqueous Sucrose Solutions up to the Pressure of 760 mm . 

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A method of determining the vapour pressure of pure sucrose solutions at relatively high temperatures is described. The design of the apparatus affords accurate control of the weight composition of the solution throughout the measurements and avoids errors inherent in earlier methods.

Systematic results covering a wide range of concentration and temperature are recorded. An empirical equation, relating vapour pressure to temperature, and enabling the elevation of the boiling point to be calculated, has been derived.

Although various investigators (Robinson and Sinclair, J. Phys. Chem., 1933, 37, 495; J. Amer. Chem. Soc., 1934, 56, 1830; Scatchard, Hamer, and Wood, ibid., 1938, 60, 3061) have used the isopiestic method for the determination of the activities of sucrose solutions at relatively low temperatures, the method has not been applied at higher temperatures. Attempts to determine systematically and accurately the vapour pressures of concentrated solutions at higher temperatures appear to be restricted to the work of Spengler, Böttger, and Werner (Z. Wirtsch. Zückerind., 1938, 88, 521). Neither the work of Perman and Saunders (Trans. Faraday Soc., 1923, 19, 112) nor that of Downs and Perman (ibid., 1927, 23, 95) covers concentrations higher than $76 \%$, i.e., 76 g . of sucrose in 100 g . of solution, nor does either supply data for any one solution over a comprehensive range of temperature. Spengler's interest is centred mainly on the technically important question of the relation between the elevation of the boiling point and the pressure as read on the manometer. He appears not to have made adequate allowance for the superheating and other sources of error in the modified Beckmann method employed by him. Notwithstanding this, his experiments give results which, in general, appear to be concordant among themselves.

We have endeavoured to determine the vapour pressures of sucrose solutions at various temperatures and concentrations by a method designed to be capable, in favourable conditions, of rigid measurement of these quantities. This method depends on the attainment of a true static equilibrium.

## Experimental.

In the course of this work, we have several times had occasion to modify the apparatus. The present form is shown in Fig. 1. It consists essentially of a flask $A$ containing the solution to be investigated, connected with one limb of the manometer $B$, the other limb of which is connected with the barometer. The pressure in the barometer can be adjusted to be equal to the vapour pressure in flask $A$ by admitting or withdrawing air through the two-way tap $M$ until the mercury is at the same level in the two arms of the manometer. To give freedom of movement, the manometer is connected through the train of tubes to the barometer by means of thick, water-jacketed pressure tubing, and is immersed in a rocking thermostat $C$. This is provided with a glass window $W$ and is kept at a temperature about $1^{\circ}$ higher than that of the main thermostat. By means of the air reservoir $L$ under adjustable mercury pressure, the apparatus is adapted for use at pressures somewhat higher than atmospheric. The arrangement shown in Fig. la, consisting of a strong metal T-piece working in a glass socket, permits of free rotation of the tap attached to cork $D$, while the springs prevent leakage under excess internal pressure. For temperatures up to about $90^{\circ}$, the liquid used in the thermostats is water, covered, in the main thermostat, by crude heavy paraffin. For higher temperatures, dry crude paraffin is used in the main thermostat and refined paraffin in the rocking thermostat.

The Precision Apparatus.-The working thermometer, graduated in $0 \cdot 1^{\circ}$, is calibrated at intervals of $5^{\circ}$ against a National Physical Laboratory standard of which the limit of accuracy is $\pm 0.02^{\circ}$. Besides being independently calibrated in the neighbourhood of $100^{\circ}$ against the b . p. of water at known atmospheric pressure, its readings have been checked in the vapour-pressure apparatus by the determination of the vapour pressure of water at intervals of $5^{\circ}$ from $60^{\circ}$ upwards. The Reichsanstalt values for the temperatures corresponding to the different vapour pressures are taken as correct. Each thermometer reading in the apparatus is corrected for exposed stem. The barometer is enclosed in a water jacket, the temperature of which is read immediately after the pressure. Both the barometer and manometer readings are corrected for pressure towards the centres of curvature of the mercury menisci. A cathetometer is used for the reading of the mercury levels. The barometer readings are corrected for temperature and also for the effect of latitude on gravity. The glass barometer scale has been compared with a Zeiss scale over lengths of 100 mm . and the average divergence is found to be 0.07 mm .

In the earlier experiments, owing to the slight up and down temperature swing of $0.03-0.08^{\circ}$, causing a corresponding swing in the mercury levels in the manometer, the temperature was read at the instant that the mercury became level in the arms of the manometer. This necessitated two operators. In the later observations at higher temperatures, giving pressures approximately atmospheric, the use of an improved thermoregulator containing chlorobenzene greatly reduces this temperature variation and enables one operator (H. C. E.) to carry on the work. Increased accuracy is also attained by the use of a short-range N.P.L. calibrated thermometer, graduated in $0.05^{\circ}$ and especially designed to afford satisfactory immersion in the thermostat.

Outline of Method.-Tate and Lyle's purest sucrose containing on a dry basis $99.99 \%$ of sucrose and $0.001-0.002 \%$ of invert sugar is used throughout. After very fine grinding to accelerate dissolution, the sucrose is dried at $60^{\circ}$ under 5 cm . pressure in a stream of dry air. It is weighed into flask $A$, which

Fig. 1.
Apparatus for determination of the vapour pressure of solutions up to atmospheric pressure.

$X X=$ Water level in thermostat.
is then sealed on to the manometer, and the requisite quantity of water is transferred from a weight burette into tube $G$, the whole of the mercury in the manometer being in reservoir $P$.

After tube $G$ has been sealed, the water is immediately frozen by means of liquid air, and the apparatus exhausted by a Cenco Hyvac and a mercury-vapour pump, till no air can be detected by the McLeod gauge. During the pumping out of the air, tube $H$ must be immersed in liquid air to detect and prevent any loss of water.

After complete evacuation, the mercury is pushed up into the manometer, thus isolating flask $A$ from the water tubes. By repeated distillation from tube $G$ to tube $H$ and vice versa, and subsequent exhaustion when the water is frozen, the air can be removed from the water, which should be as free as possible from carbon dioxide before being put into the apparatus. Conductivity water is used, but as a final precaution, a few c.c. of water may be distilled over into tube $J$, which is then sealed off, and the water discarded after its weight has been determined. The mercury in the manometer is drawn down, the flask $A$ immersed in carbon dioxide and methylated spirit, and the water is distilled into the sucrose. The flask is again closed by pushing the mercury up into the manometer, and the sugar is dissolved by rocking the thermostat $C$. With solutions of high concentration, it is preferable to obtain the requisite high temperature for solution by rapidly heating the flask in a large beaker, containing,
e.g., calcium chloride solution, and then replacing the beaker by the heated main thermostat. The thermostat $C$ is continually rocked until equilibrium is attained, effective disturbance of the solution being ensured by the marbles $R$, which project through the surface. The mercury in the manometer is levelled, and the shaking stopped only for the time required to take the temperature reading. At least ten such readings are rapidly taken and the mean is the equilibrium temperature. The variation in the mean reading for duplicate sets of observations at any one temperature is not more than $0.02^{\circ}$. Any drift in the readings is taken to indicate that equilibrium has not been attained, and the process is repeated after further shaking.

Fig. 2.
Graphic relationship between $\log _{10} \mathrm{p}_{8}$ and $10^{3} / T$.
$t^{\circ} \mathrm{C}$.


- With the exception of this point, the values of $\mathrm{A}^{\prime}$ coincide too closely with those of A to permit insertion in the graph.

As an additional precaution, at least two independent readings are taken at each round temperature, and in the one case equilibrium is attained by lowering, and in the other by raising the temperature, thus lessening the possibility of unrecognised errors caused by the presence of condensed drops of solvent or of surface layers differing in concentration from the main liquid. Since inversion of sucrose is known to occur in aqueous solution at a rate depending on the temperature as well as on other factors, the readings are taken at temperatures which are not consecutive. Appreciable and gradually increasing inversion would, therefore, produce irregularities in the pressure-temperature curve, rather than a steady drift which might escape observation.

At the completion of the vapour-pressure measurements with any one solution, the total weight is redetermined to ensure that no loss of water has occurred. Any change in weight has, in every case, been found to be negligible, being of the order of $0.01 \%$ of the total weight of the solution. In calculating the coucentration of the solutions no correction for buoyancy of air is applied. The weight of water forming the vapour phase is subtracted from the total weight of water. This correction is
equivalent to an addition of $0.03-0.07$ to the percentage of sucrose in solution, depending on the temperature, pressure, and dimensions of the apparatus, as well as on the concentration and total volume of the solution. At the end of every series of experiments with any one solution, the latter is examined by the polarimeter to determine the percentage of sucrose inverted. Also, in cases in which the solution has been subjected to higher temperatures, such as those reached in the determination of vapour pressures, of the order of one atmosphere, the percentage of invert present is determined by Lane and Eynon's method to detect possible further breakdown of invert sugar. In some such cases, Clerget's method has also been used.

The experimental work falls into two parts. The first deals with solutions examined over a wide range of pressure, at temperatures in no case exceeding $95^{\circ}$; the second part is mainly concerned with the temperatures required to produce a vapour pressure of 760 mm ., i.e., with the elevation of the b. p. under normal atmospheric pressure.

## Results.

## Part I.

Table I records the observed temperatures, $t^{\circ}$, and the corresponding vapour pressures, $p_{s}$ (in mm .), for sucrose solutions, in which the weight concentration ranges from about 45 to $83 \%$, and the mole-fraction of the sucrose, $N_{2}$, from about 0.04 to 0.20 . The table also gives the elevation of the boiling points, $E_{\text {exp. }}$. calculated from these pressures and temperatures, the requisite b. p.s of water being taken from the Reichsanstalt Tables (Landolt-Börnstein, 1931, Erg. IIb, 1329 et seq.). $E_{\text {calc. }}$. refers to values of $E$ smoothed by means of equation (2) (see below). At the foot of the column for each solution is recorded the percentage of the initial sucrose found to be inverted at the end of the experiment. These symbols are retained throughout the paper.

Fig. 2 shows, for the different solutions, the relationship between $\log _{10} p_{s}$ and $10^{3} / T$, where $T$ is the absolute temperature ( $0^{\circ} \mathrm{c}=273 \cdot 16^{\circ} \mathrm{k}$.), corresponding to the observed vapour pressure $p_{s}$. The left-hand portion of the diagram is the continuation upwards of the righthand portion and, for reading, should be superimposed on the latter as indicated by the marginal temperatures and pressures. The vapour pressure curve for water is on the extreme right of each portion of the diagram. On the scale of the original drawing ( $21^{\prime \prime} \times 17^{\prime \prime}$ ), these curves do not deviate appreciably from straight lines and are very nearly parallel to each other. The slopes of the lines give the heats of vaporization of water from the solutions. Any deviation from exact parallelism indicates that there is a heat of dilution.

For each solution, values of $\log _{10} p_{s} / p_{w}$ ( $p_{w}$ is the vapour pressure of water at the temperature $T$ ) have been calculated for each temperature and plotted against $T^{-1}$. It is quite clear from these curves that $\log _{10} p_{s} / p_{w}$ depends on the temperature and hence that there is a heat of dilution, $\Delta \bar{H}_{1}$, i.e.,

$$
\begin{equation*}
\frac{\partial \log _{10} p_{s} / p_{w}}{\partial\left(T^{-1}\right)}=\frac{-\Delta \bar{H}_{1}}{2 \cdot 303 \boldsymbol{R}} \tag{1}
\end{equation*}
$$

In order to smooth the results and obtain an empirical equation for them, the following procedure was adopted. For each of the solutions, mean squares were used to obtain a relation between $\log p_{s} / p_{w}$ and $T^{-1}$ of the form

$$
\log p_{s} / p_{w}=A\left(N_{2}\right)+B\left(N_{2}\right) / T
$$

where $A$ and $B$ are taken as independent of the temperature, but functions of $N_{2}$, the molefraction of the sucrose. Then the values of $\log p_{s} / p_{w}$ were calculated for the respective solutions at $80^{\circ}$ (approximately the median temperature for all solutions). These values at $80^{\circ}$ were smoothed by applying mean squares to find the constants $a$ and $b$ in the equation

$$
\frac{1}{N_{2}}\left(\log _{10} \frac{p_{w}}{p_{s}}\right)_{80}-0 \cdot 4343=a N_{2}+b N_{2}{ }^{2}
$$

The values of $B\left(N_{2}\right)$ were plotted against $N_{2}$ and it was found that, within the limits of the experimental accuracy, they were proportional to $N_{2}{ }^{2}$. Finally, the results were assembled into an empirical relation

$$
\begin{equation*}
\log _{10} p_{w} / p_{s}=0.4343 N_{2}+(0.4721+713 / T) N_{2}{ }^{2}-1.32 N_{2}{ }^{3} \tag{2}
\end{equation*}
$$

which may be considered to represent our results for the vapour pressures of sucrose solutions from 45 to $85 \%$ concentration between the temperatures of $60^{\circ}$ and $95^{\circ}$.

Calculations from this equation of $p_{s}$, for comparison with the experimental results, show close agreement. The degree to which it represents the experimental data may be judged from the values of $E_{\text {calc., }}$, the calculated elevation of the b. p., which are given in Table I.

## Table I.

## The vapour pressures of pure sucrose solutions at various temperatures.

$N_{2}=$ mole-fraction of sucrose; $p_{1}=$ vapour pressure of the solution in $\mathrm{mm} . \mathrm{Hg}$ at temperature $t^{\circ} \mathrm{C} ; E_{\text {exp. }}=$ elevation of $\mathrm{b} . \mathrm{p}$. calculated from $p_{s}$ and $t^{\circ}$ and the b . p . of water under pressure $p_{\mathrm{a}}$ : $E_{\text {call. }}=$ elevation of b. p. calculated from equation (2).

|  | A |  |  |  | $\mathrm{A}^{\prime}$ |  |  |  | B |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | g . of sucrose per 100 g . of solution. |  |  |  |  |  |  |  |  |  |  |  |
|  | 44.85 |  |  |  | 44.87 |  |  |  | 58.90 |  |  |  |
| $N_{2}$ | $0 \cdot 041043$ |  |  |  | 0.041077 |  |  |  | 0.070135 |  |  |  |
| Temp. region | $E \text {, }$ |  |  |  | $\overbrace{}^{E,}$ |  |  |  |  |  | $\underbrace{E}$ |  |
|  | $t,{ }^{\circ} \mathrm{C}$ | $p$ 。 | exp. | calc. | $t,{ }^{\circ} \mathrm{C}$ | $p$, | exp. | calc. | $t,{ }^{\circ} \mathrm{C}$ | $p$ s | exp. | calc. |
| $60^{\circ}$ | - | - | - | - | $60 \cdot 20$ | 143.145 | 1-12 | $1 \cdot 10_{6}$ | $60 \cdot 17$ | 136.02 | $2 \cdot 18$ | $2 \cdot 12$ |
|  |  |  |  |  |  | 177.75 |  | - | $60 \cdot 40$ 64.88 | 137.63 169.37 |  | - |
| 65 | $65 \cdot 09$ | 179.38 | 1.08 | $1 \cdot 13$ | $64 \cdot 89$ | 177.75 | 1.08 | 二 | 64.88 | 169.37 | $2 \cdot 14$ | 2.18 |
|  | $65 \cdot 26$ $70 \cdot 08$ | 180.68 222.77 | 1.09 1.18 | $1.13{ }_{4}$ 1.165 | $65 \cdot 21$ $70 \cdot 07$ | 180.06 222.95 | 1.12 | - | $65 \cdot 24$ $70 \cdot 10$ | $171 \cdot 73$ 213.00 | $2 \cdot 20$ $2 \cdot 23$ | $2 \cdot 18$ $2 \cdot 25$ |
| 70 | $70 \cdot 39$ | $226 \cdot 16$ | $1 \cdot 14$ |  | $70 \cdot 46$ | $227 \cdot 05$ | $1 \cdot 12$ | - | $70 \cdot 21$ | $213 \cdot 57$ | $2 \cdot 27$ |  |
| 75 | 74.98 | 274.70 | $1 \cdot 19$ | $1 \cdot 204$ | 74.98 | 274-18 | $1 \cdot 24$ | - | $74 \cdot 89$ | $260 \cdot 86$ | $2 \cdot 32$ |  |
|  | $75 \cdot 29$ | 277-76 | 1.24 | - | 75.34 | $278 \cdot 61$ | 1.22 | - | $75 \cdot 30$ | $265 \cdot 02$ | $2 \cdot 36$ | $2 \cdot 31$ |
| 80 | 80.08 | 338.90 | $1 \cdot 23$ | - | $79 \cdot 49$ | $338 \cdot 35$ | $1 \cdot 18$ | - | 80.07 | 323.25 | $2 \cdot 37$ | $2 \cdot 38$ |
|  | 80.29 | 341.61 | 1.24 | $1 \cdot 24{ }_{6}$ | 80.21 | $340 \cdot 90$ | 1.22 | - | 80.36 | $327 \cdot 18$ | $2 \cdot 37$ |  |
| 85 | 84.97 | $412 \cdot 38$ | 1.24 | - | 84.73 | 407.99 | $1 \cdot 27$ | - | 84.77 | 389-45 | $2 \cdot 47$ | $2 \cdot 47$ |
|  | $85 \cdot 37$ | 417.80 | 1.31 | $1 \cdot 27{ }_{6}$ | 85.07 | 413.09 | $1 \cdot 30$ | - | 85.04 | 394-21 | $2 \cdot 44$ |  |
| 90 | 89.86 | $498 \cdot 28$ | $1 \cdot 27$ | - | 89.85 | $497 \cdot 70$ | $1 \cdot 29$ | - | $89 \cdot 84$ | 474-15 | $2 \cdot 54$ | - |
|  | 90.28 | 505.56 | $1 \cdot 31$ | $1 \cdot 32{ }_{6}$ | $90 \cdot 27$ | $505 \cdot 67$ | 1.29 | - | $90 \cdot 18$ | $480 \cdot 69$ | $2 \cdot 54$ | 2.53 |
| 95 | 94.89 | $601 \cdot 29$ | 1.32 | $1 \cdot 36$ | 94.47 | $592 \cdot 83$ | 1.28 | - | $95 \cdot 10$ | 578.59 | $2 \cdot 56$ | $2 \cdot 60$ |
|  | $95 \cdot 14$ | 607.33 | $1 \cdot 30$ | - | $95 \cdot 36$ | 612.96 | $1 \cdot 29$ | - | $95 \cdot 46$ | 586.97 | $2 \cdot 53$ | - |
|  | $1 \cdot 17$ * |  |  |  | $2 \cdot 21$ |  |  |  | 1.07 |  |  |  |
| C |  |  |  |  | D |  |  |  | E |  |  |  |

g . of sucrose per 100 g . of solution.


* Data in the last line of each section record the sucrose inverted as g. per 100 g . of original.

Table I-continued.


* For solution H, data are as follows: $82.22 \mathrm{~g} . / 100 \mathrm{~g}$. of solution, $N_{2}=0.193741$, sucrose inverted 0.39 .

$$
\begin{array}{ccccccc}
t,{ }^{\circ} \mathrm{C} & P_{s} & E_{\text {exp. }} & E_{\text {calc. }} & t,{ }^{\circ} \mathrm{C} & P_{s} & E_{\text {exp. }} \\
95.00 & 432 \cdot 10 & 10.09 & 10 \cdot 19_{4} & 95 \cdot 35^{\circ} & 437.76 & 10 \cdot 11
\end{array}
$$

By means of this equation, the smoothed values of $p_{s}, E$, and $\log _{10} p_{w} / p_{s}$ have been calculated for $45-90 \%$ concentrations at $5 \%$ intervals and for $60-95^{\circ}$ at $5^{\circ}$ intervals. These values are listed in Table II.

## Table II.

$V$ alues of $\log _{{ }_{10}} \mathrm{p}_{\mathrm{w}} / \mathrm{p}_{\mathrm{s}}, \mathrm{p}_{\mathrm{s}}$, and E , calculated by means of equation (2), in which $\mathrm{p}_{\mathrm{w}}$ and $\mathrm{p}_{\mathrm{s}}$ represent the vapour pressure of water and of the solution, respectively, at the temperature $\mathrm{t}_{0}$, and E represents the elevation of the boiling point at the same temperature.

| g. sucrose/ 100 g. soln. | 45 | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $10^{7} N_{2}$ | 412,839 | 731,698 | 890,396 | 1,093,737 | 1,363,621 | 1,739,111 | 2,297,273 | 3,214,257 |
| $t_{0}$ | $10^{7} \times \log _{10} p_{w} / p_{s}$ |  |  |  |  |  |  |  |
| $60^{\circ}$ | 222,887 | 452,454 | 584,477 | 770,222 | 1,044,493 | 1,475,920 | 2,216,244 | 3,656,379 |
| 65 | 222,348 | 450,761 | 581,969 | 766,436 | 1,038,609 | 1,466,350 | 2,199,546 | 3,623,690 |
| 70 | 221,813 | 449,116 | 579,533 | 762,761 | 1,032,907 | 1,457,058 | 2,183,333 | 3,591,949 |
| 75 | 221,300 | 447,571 | 577,165 | 759,192 | 1,027,348 | 1,448,033 | 2,167,585 | 3,561,121 |
| 80 | 220,821 | 445,966 | 574,876 | 755,723 | 1,021,957 | 1,439,264 | 2,152,283 | 3,531,167 |
| 85 | 220,340 | 444,457 | 572,634 | 752,352 | 1,016,716 | 1,430,757 | 2,137,409 | 3,502,047 |
| 90 | 219,774 | 442,990 | 570,417 | 749,073 | 1,011,620 | 1,422,450 | 2,122,946 | 3,473,731 |
| 95 | 219,419 | 441,563 | 568,347 | 745,883 | 1,006,662 | 1,414,386 | 2,108,873 | 3,446,183 |
| $p_{z}, \mathrm{~mm}$. |  |  |  |  |  |  |  |  |
| 60 | $141 \cdot 907$ | 134.601 | $130 \cdot 570$ | 125-104 | $117 \cdot 447$ | $106 \cdot 349$ | $89 \cdot 674$ | $64 \cdot 366$ |
| 65 | $178 \cdot 180$ | 169.051 | 164.020 | 157.199 | $147 \cdot 649$ | 133.801 | $113 \cdot 016$ | 81.419 |
| 70 | $220 \cdot 636$ | $210 \cdot 740$ | 204.506 | $196 \cdot 057$ | 184.232 | $167 \cdot 091$ | $141 \cdot 360$ | $102 \cdot 203$ |
| 75 | 274.738 | $260 \cdot 790$ | 253-123 | 242-733 | 228-199 | 207-131 | 175-505 | 127-332 |
| 80 | 337-496 | $320 \cdot 445$ | 311.074 | $298 \cdot 386$ | $280 \cdot 644$ | 254.932 | 216-332 | $157 \cdot 483$ |
| 85 | $412 \cdot 154$ | $391 \cdot 421$ | $380 \cdot 037$ | 364-631 | $343 \cdot 098$ | 311.899 | $265 \cdot 063$ | 193-591 |
| 90 | $499 \cdot 816$ | 474.766 | 461.048 | $442 \cdot 466$ | 416.510 | 378.916 | 322.473 | $236 \cdot 273$ |
| 95 | $602 \cdot 668$ | 572-674 | $556 \cdot 143$ | $533 \cdot 866$ | $502 \cdot 753$ | 457.701 | $390 \cdot 062$ | $286 \cdot 683$ |
| $E^{\circ}, \mathrm{c}$ |  |  |  |  |  |  |  |  |
| 60 | $1 \cdot 10_{5}$ | $2 \cdot 23{ }_{3}$ | $2 \cdot 880$ | $3 \cdot 78{ }_{1}$ | $5 \cdot 10_{8}$ | $7 \cdot 16_{4}$ | $10 \cdot 62_{4}$ | $17 \cdot 12_{8}$ |
| 65 | $1 \cdot 14{ }_{0}$ | $2 \cdot 303$ | $2 \cdot 967$ | $3 \cdot 89_{3}$ | $5 \cdot 25{ }_{2}$ | $7 \cdot 360$ | $10.90_{6}$ | $17 \cdot 55_{4}$ |
| 70 | $1 \cdot 17{ }_{2}$ | $2 \cdot 36{ }_{9}$ | $3 \cdot 05_{1}$ | $4 \cdot 00_{4}$ | $5 \cdot 39{ }_{6}$ | $7 \cdot 559$ | $11 \cdot 188$ | 17.977 |
| 75 | $1 \cdot 214$ | $2 \cdot 43{ }_{7}$ | $3 \cdot 13_{4}$ | $4 \cdot 11_{7}$ | $5 \cdot 54{ }_{6}$ | $7 \cdot 76{ }_{1}$ | $11.47{ }^{5}$ | $18 \cdot 40_{8}$ |
| 80 | $1 \cdot 25{ }^{\text {a }}$ | $2 \cdot 518$ | $3 \cdot 240$ | $4 \cdot 24_{4}$ | $5 \cdot 70_{5}$ | 7.970 | $11.77{ }_{1}$ | $18.85{ }_{4}$ |
| 85 | $1 \cdot 28{ }_{4}$ | $2 \cdot 58$ | $3 \cdot 31{ }_{8}$ | $4 \cdot 345$ | $5 \cdot 850$ | $8 \cdot 17{ }_{7}$ | $12 \cdot 06{ }_{2}$ | 19.288 |
| 90 | $1 \cdot 32{ }_{6}$ | $2 \cdot 66{ }_{3}$ | $3 \cdot 42{ }_{5}$ | $4 \cdot 48{ }_{4}$ | $6 \cdot 018$ | $8 \cdot 392$ | $12 \cdot 364$ | $19 \cdot 74_{3}$ |
| 95 | $1 \cdot 36{ }_{6}$ | $2 \cdot 73{ }_{4}$ | $3 \cdot 51{ }_{2}$ | $4 \cdot 597$ | $6 \cdot 178$ | $8 \cdot 617$ | $12 \cdot 66_{8}$ | $20 \cdot 20{ }_{1}$ |

Also, by a method of approximations, equation (2) has been employed to calculate at the above regular intervals of concentration, the elevations of the $b$. $p$. under fixed pressures. Table III records these elevations for 760 mm . and the stated fractions of 760 mm . pressure.

Table III.
Elevations of the boiling point, $\mathrm{E}^{\circ}$, calculated by means of equation (2) for sucrose solutions of concentration C (mole-fraction of sucrose $=\mathrm{N}_{2}$ ) under pressure $\mathrm{p}_{\mathrm{s}}(\mathrm{mm})$ ).

| $p_{s}, \mathrm{~mm}$. |  | 190 | 380 | 570 | 760 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Concn. } C \text { (g. } \\ \text { sucrose } / 100 \mathrm{~g} \text {. soln.) } \end{gathered}$ | $N_{2}$ | Elevation of boiling point, $E^{\circ}$, under pressure $p_{s}$ (mm.) |  |  |  |
| 45 | 0.0412839 | $1 \cdot 150$ | 1.27 \% | $1.35{ }_{4}$ | $1.41{ }_{6}$ |
| 60 | 0.0731698 | $2 \cdot 33{ }_{8}$ | $2.57{ }_{2}$ | $2.73{ }_{5}$ | $2 \cdot 85{ }_{8}$ |
| 65 | 0.0890396 | $3 \cdot 02$ | $3.31{ }_{8}$ | 3.526 | $3 \cdot 68{ }^{\text {2 }}$ |
| 70 | $0 \cdot 1093737$ | 3.991 | $4 \cdot 36{ }_{5}$ | 4.63 , | $4 \cdot 84_{1}$ |
| 75 | $0 \cdot 1363621$ | $5 \cdot 414$ | $5 \cdot 928$ | $6 \cdot 28{ }_{0}$ | $6.54{ }_{9}$ |
| 80 | $0 \cdot 1739111$ | $7.68{ }_{1}$ | $8 \cdot 39{ }_{5}$ | $8.86{ }_{4}$ | $9 \cdot 23{ }_{3}$ |
| 85 | $0 \cdot 2297273$ | 11.581 | $12.62{ }^{5}$ | $13 \cdot 30{ }_{7}$ | $13 \cdot 84{ }_{8}$ |
| 90 | $0 \cdot 3214257$ | $19 \cdot 24{ }_{9}$ | $20 \cdot 88$ 8 | $21.97{ }_{3}$ | $22 \cdot 81{ }_{2}$ |

Table III enables the calculated results to be compared with those of earlier investigators and with the results of the actual determinations in Tables I and V.

Further, with regard to the application of equation (2), it should be pointed out that the heat of dilution $\Delta H$ can be obtained from the coefficient of $T^{-1}$. This corresponds to the relation

$$
\begin{equation*}
\Delta \bar{H}_{1}=-3.3 \times 10^{3} N_{2}{ }^{2} \text { cals } . / \mathrm{mol} \tag{3}
\end{equation*}
$$

and indicates that the heat of dilution is small. Direct measurements of the heat of dilution of sucrose have been made previously (Stackelberg, Z. physikal. Chem., 1898, 26, 533; Wood, Trans. Faraday Soc., 1917, 13, 123; Hunter, ibid., 1926, 22, 194; Vallender and Perman, ibid., 1931, 27, 126; Gucker, Pickard, and Planck, J. Amer. Chem. Soc., 1939, 61, 459), the results of Vallender and Perman being the most extensive. Although our relation (3) should be considered only as a term in an empirical expression for our vapour pressures, yet it is interesting to compare the value which Vallender and Perman obtained for the heat of dilution of $68.79 \%$ sucrose at $80^{\circ}$ with the value obtained for (3). As regards temperature and concentration, this lies close to the median of our ranges of concentration and temperature. For a $68.79 \%$ solution ( $N_{2}=0.10394$ ) equation (3) gives -1.98 cals./g.; direct determination by Vallender and Perman gave -2.02 cals./g. This close agreement must be regarded as merely fortuitous.

Inversion.-The sucrose used throughout these investigations contains, initially, not more than $0.002 \%$ of invert sugar. During use in the vapour-pressure apparatus, this proportion increased to not more than $2.21 \%$ of the original weight of sucrose (see Table I). The extent of inversion depends on many factors, such as concentration, temperature, pH , and time of heating, and, in the case of pure sucrose solutions, it may be complicated by the presence of an induction period. The detailed investigation of inversion is outside the scope of the present work, but the following experiments provide some information on the condition of the solutions at various times during the consecutive vapour-pressure measurements.

A $44.92 \%$ solution, made up with water of conductivity $1 \times 10^{-6} \mathrm{mho}$, is heated in vacuo for successive periods of 2 hours at $60^{\circ}, 65^{\circ}, 70^{\circ}, 75^{\circ}$, and $80^{\circ}$. In the course of the experiment, the solution is left to cool overnight several times. The polarimeter reading is the same at the end as at the beginning, indicating that no change in concentration and no inversion measurable by this method has taken place. However, after subsequent consecutive twohour periods of heating at $90^{\circ}, 95^{\circ}$, and $100^{\circ}$, the percentages of sucrose inverted are 0.32 , $0 \cdot 76$, and $3 \cdot 0$, respectively. After further heating for 13 hours at $100^{\circ}, 99 \%$ of the sucrose present is found to be inverted.

In the case of solution " $E$," after the vapour pressure readings given in Table I had been taken up to $85^{\circ}$, the solution was kept at about $85^{\circ}$ for a further 30 hours. After this prolonged heating, in all about 50 hours, vapour-pressure readings were taken at intervals of $\mathbf{1 - 2}$ hours, as shown in Table IV.

Owing to this inversion, $N_{1}$, the mole-fraction of the water in the solution, is reduced from 0.8284 to 0.8075 , and $N_{2}$, the mole-fraction of the solute (sucrose and invert sugar) is increased to $0 \cdot 1925$. By calculation from equation (2), it is found that at this mole-fraction, an unin-

## Table IV.

Solution E. Concentration: 79.74 g . of sucrose per 100 g . of solution: $N_{2}=0.171610$.
$h \quad=$ total hours of heating before the readings are taken.
$t_{0}{ }^{\circ} \quad=$ temperature of the solution under vapour pressure $p_{s}$.
$E_{\text {exp. }}=$ observed elevation of b. p.
$E_{\text {calc. }}$. $=$ elevation calculated from equation (2), inversion being assumed not to have occurred.
$\left.\begin{array}{ccccc}h & t_{0} & p_{s} & E_{\text {exp. }} & E_{\text {cale. }} \\ \mathbf{5 4} & 82 \cdot \mathbf{4 0} & 20^{\circ} & 274.44 & 8.64 \\ 53 & 82.58 & 276.64 & 8.63 & \mathbf{7 . 9 2 5} \\ 48 & 87.46 & 339.33 & 8.58 \\ 49 & 87.64 & 340.98 & 8.64 & \\ 51 & 92.40 & 408.89 & 8.89 & \\ 50 & 92.82 & 416.12 & 8.87\end{array}\right\}$

Percentage of initial sucrose inverted after 54 hours $=12.2$.
verted sucrose solution would show elevations of $9.39^{\circ}, 9.54^{\circ}$, and $9.85^{\circ}$ respectively at the mean temperature of each pair of readings. The elevations $E_{\text {exp. }}$ found for the partly inverted solute (Table IV, col. 4) are considerably less, indicating that the osmotic properties of sucrose solutions containing some invert sugar are not simply a function of the mole-fraction of the water, but depend also on the sizes of the molecules present. In the present case, it is fortunate that the invert sugar has not such a marked effect on the osmotic properties as one might expect from the reduction produced in the water mole-fraction, and we may assume that the vapour pressures recorded in Table I are not appreciably affected by the small amounts of inversion which have occurred. Nevertheless, in all work on sucrose solutions, it is obviously important to keep a strict check on inversion, to reduce as much as possible the time of exposure to high temperature, and to ensure the purity of the water. There is, however, some indication that the presence of even a small percentage of invert sugar tends to prevent the crystallisation of supersaturated solutions, thereby facilitating the determination of their vapour pressures. For example, several attempts to obtain an approximately $83 \%$ solution which was stable enough to give satisfactory vapour-pressure readings below $95^{\circ}$, were unsuccessful. In each of these cases, the times of heating (at $105-108^{\circ}$ ) to effect solution had been curtailed as much as possible, and the final inversion was found to be less than $1 \%$ (see Table I , solution $J$, concentration $83.22 \%$, final inversion, $0.6 \%$ of the sucrose). Solution $J_{2}$, of appreciably the same composition as $J_{1}$, was then prepared by maintaining the high temperature of solution for a considerably longer period. By this means, consistent readings were obtained at $5^{\circ}$ intervals to the $80^{\circ}$ region. The solubility of sucrose at $80^{\circ}$ is 78.805 g . per 100 g . of solution (Taylor, $J ., 1947,1682$ ). The final inversion was, in this case, found to be $1.01 \%$ of the total sucrose.

Since temperature is one of the more important factors influencing the rate of inversion, the vapour-pressure determinations in the foregoing experiments have been limited to temperatures not exceeding $95^{\circ}$. In concentrated solutions, temperatures considerably higher than $95^{\circ}$ are required to produce a vapour pressure of 760 mm . The determination of these higher temperatures (i.e., the b. p.s of concentrated sucrose solutions under 760 mm .) by the vapour-pressure method has been carried out independently by one of us (H. C. E). The experimental work and results are described in Part II.

## Part II.

A preliminary experiment showed that, after being heated for 6 hours under a vapour pressure of approximately 760 mm ., a $60 \%$ sucrose solution had undergone inversion to the extent of $20 \%$. Therefore, in order to reduce the time of exposure at high temperatures, the vapour pressure of each solution is determined at only two temperatures, one slightly above and the other slightly below that required to give a vapour pressure of 760 mm . The b. p. under 760 mm . is then determined by interpolation in the logarithmic curve connecting the experimental points (Fig. 2). By this means, the total time of heating is reduced to about 3 hours, and the percentage, in the different solutions, of sucrose inverted, as determined by the polarimeter, is found to lie between 3.71 and $0.82 \%$. The direct determination of invert sugar by Lane and Eynon's method gives the corresponding percentages as 3.61 and $0.87 \%$, respectively, showing that no significant molecular breakdown other than inversion has occurred.

Table V records, for each of the solutions, the concentration, the actual vapour pressure data, and the interpolated temperature, $t_{\text {int., }}$, at which $p_{s}=760 \mathrm{~mm}$. Cols. 5 and 6 give the resulting elevation of the b. p. at $760 \mathrm{~mm} ., E_{\text {exp., }}$ and the elevation, $E_{\text {calc., }}$, as calculated from
equation (2) and the vapour pressure tables for water given in Landolt-Börnstein's "Tabellen " (1936, Vol. IIIc, p. 2415). The complete inversion data are collected in cols. 7 and 8.

## Table V.

Elevation of boiling point of pure sucrose solutions at 760 mm . pressure.
$p_{s} \quad=$ vapour pressure in mm. at $t_{0}{ }^{\circ}$.
$t_{\text {int. }}=$ interpolated temperature at 760 mm .
$E_{\text {exp. }}=$ elevation of b. p. at 760 mm .
$E_{\text {calc. }}=$ elevation of b. p. at 760 mm . calculated from eqn. (2).

| Concn., \% | $p$ s | $t_{0},{ }^{\circ} \mathrm{C}$ | $t_{\text {int }}$. | $E_{\text {exp. }}, 760 \mathrm{~mm}$. | $E_{\text {calc. }}, 760 \mathrm{~mm}$. | $I_{1}{ }^{*}$ | $I_{2}{ }^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $43 \cdot 23$ | $\begin{aligned} & 747 \cdot 22 \\ & 763 \cdot 31 \end{aligned}$ | $\begin{aligned} & 100.77 \\ & 101.38 \end{aligned}$ | 101.25 | 1.25 | $1 \cdot 30_{2}$ | $1 \cdot 29$ | 1.62 |
| $60 \cdot 12$ | $\begin{aligned} & 758.29 \\ & 770.59 \end{aligned}$ | $\begin{aligned} & 102.77 \\ & 103.22 \end{aligned}$ | 102.83 | $2 \cdot 83$ | $2 \cdot 88{ }_{8}$ | $1 \cdot 60$ | - |
| $69 \cdot 27$ | $\begin{aligned} & 756.39 \\ & \mathbf{7 6 7 . 9 7} \end{aligned}$ | $\begin{aligned} & 104.69 \\ & 105.06 \end{aligned}$ | 104•79 | $4 \cdot 79$ | $4 \cdot 66_{6}$ | 3.71 | $3 \cdot 61$ |
| 75.08 | $\begin{aligned} & 751.75 \\ & 763.90 \end{aligned}$ | $\begin{aligned} & 106.45 \\ & 106.89 \end{aligned}$ | 106.75 | 6.75 | $6 \cdot 60_{7}$ | $2 \cdot 23$ | $2 \cdot 25$ |
| 79.08 | $\begin{aligned} & 755 \cdot 19 \\ & 763 \cdot 05 \end{aligned}$ | $\begin{aligned} & 108.68 \\ & 108.94 \end{aligned}$ | 108.84 | $8 \cdot 84$ | $8 \cdot 66{ }_{2}$ | 2.78 | 2.29 |
| 82.74 | $\begin{aligned} & 750 \cdot 17 \\ & 772 \cdot 45 \end{aligned}$ | $\begin{aligned} & 111.06 \\ & 111.95 \end{aligned}$ | 111.45 | 11.45 | 11.449 | $1 \cdot 04$ | 1.03 |
| 84.72 | $\begin{aligned} & 748 \cdot 07 \\ & 766 \cdot 22 \end{aligned}$ | $\begin{aligned} & 112.96 \\ & 113.65 \end{aligned}$ | 113.41 | 13.41 | 13.529 | 1.36 | 1-43 |
| 86.36 | $\begin{aligned} & 749 \cdot 28 \\ & 763 \cdot 92 \end{aligned}$ | $\begin{aligned} & 114 \cdot 94 \\ & 115 \cdot 46 \end{aligned}$ | 115.33 | 15-33 | $15 \cdot 70$ | 1.56 | 1.35 |
| 87.87 | $\begin{aligned} & 756 \cdot 28 \\ & 778 \cdot 13 \end{aligned}$ | $\begin{aligned} & 117.51 \\ & 118.24 \end{aligned}$ | 117.62 | 17.62 | $18 \cdot 18{ }_{4}$ | $0 \cdot 82$ | $0 \cdot 87$ |

$$
\begin{array}{ccc}
* I_{1}=\mathrm{g} . \text { of invert sugar produced per } 100 \mathrm{~g} . \text { of sucrose (by polarimeter). } \\
I_{2}= & , \quad " \quad \text { (by Lane and Eynon's method). }
\end{array}
$$

The generally accepted values of $E$ at 760 mm . pressure are those of Claassen ( $Z$. Ver. Deutsch. Zückerind., Techn. T., 1904, 54, 1159) and of Spengler, Böttger, and Werner (loc. cit., p. 601). In 1941, Tressler, Zimmerman, and Willits (J. Physical Chem., 45, 1242) carried out a series of boiling-point determinations under atmospheric pressure, designed to give values for the elevation of the boiling point as accurate as are obtainable by a differential modification of the original Beckmann method. The work of all these investigators suffers from the drawback that, although the total weight concentration of the solution purported to be determined at the end of each experiment, no attempt was made to determine the final compositionsucrose and invert sugar-of the solute. Claassen, making determinations of $E$ under atmospheric pressure, reached, in the case of the highest concentrations, temperatures between $120^{\circ}$ and $130^{\circ}$, and assuming no inversion, he relied on the polarimeter to give the total concentration. Owing to the opposing effects of the sucrose and invert sugar on the polarimeter reading, the unsuspected presence of even a small percentage of invert sugar would cause a relatively large error in the result. Spengler and Tressler each relied solely upon the refractometer for the same purpose; at lower and medium temperatures, the consequent error might be inconsiderable, since the correction for the presence of invert sugar in the determination of the total solid in a sucrose solution by this method is small. Also, we have demonstrated above that the effect on the vapour pressure of the conversion of sucrose into invert sugar is smaller than would be calculated from the decrease in the mole-fraction of the water in the solution.

The experiments of Tressler et al. were not carried out at 760 mm ., but under the prevailing atmospheric pressure. However, the b. p.s of the solution and of the water were determined at the same time and, since the altitude of their laboratory appears to be about 600 feet, the divergence of their values of $E$ from the values at 760 mm . might not be large.

In Spengler's experiments, the pressures were accurately measured and the values of $t_{0}$ at 760 mm . were obtained by interpolation, but, in addition to the uncertainty regarding the final percentage of invert sugar, there are well-known errors inherent in the original Beckmann method which Spengler's calibration of his apparatus with pure water (loc. cit., p. 554) would not enable him to estimate. Also, his method of establishing high pressures with solutions of high concentration is open to criticism (loc. cit., p. 552). A great advantage of the method now recorded is that it enables the solutions to be made up very accurately to a known concentration, which not only is maintained throughout the experiment, but also can be checked at the end. Then, as shown above, any one or more of the methods for the estimation of invert can be applied.

To obtain a comparison of the relative consistency of the results of the various methods described, the values of the ratio $E / N_{2}$, calculated from the results at 760 mm . of each of the authors concerned, have been plotted against the corresponding mole-fractions of the sucrose $\left(N_{2}\right)$, and the results are illustrated in Fig. 3. The continuous line has been drawn smoothly through all the results (indicated by arrows) from Table V , and the broken line corresponds to the values of $E$ calculated by means of equation (2). The agreement between these lines is quite good, especially when the extent to which the equation has been extrapolated is considered. Not only has it been extrapolated with regard to temperature, but also to some extent

Fig. 3.
Comparison of $\mathrm{E} / \mathrm{N}_{2}$ against $\mathrm{N}_{2}$ calculated from equation (2) and from the results of different investigators.

with regard to concentration, the equation having been derived from results obtained with solutions ranging in concentration from about 45 to $85 \%$ (mole-fraction $0.04-0.20$ ) and over this range it is seen that the values of $E / N_{2}$ obtained from the equation and from direct experiment agree closely. The deviation between the two appears to be due to a sigmoid tendency in the experimental curve and, naturally, the simple algebraic relation assumed for the equation cannot reflect this. The results of Tressler et al. are in good agreement with the present values except at their highest concentration-about $72 \%$ of sucrose-at which they had obviously reached the limit of concentration for which their apparatus was suited. The values given by Claassen and Spengler produce curves of a more pronounced sigmoid tendency. It is difficult to compare the results of these two authors with the present results. The relatively low values of $N_{2}$ indicated in the larger proportion of Claassen's results may be referred, as already indicated, to the use of the polarimeter alone for the final determination of concentration. On the other hand, in the case of Spengler's highest concentration, the failure of $E$ to increase consistently with $N_{2}$ may be due to the introduction of nitrogen into his apparatus.

Further work on the problems raised by the foregoing results is now in progress in this laboratory.

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